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Phenomenal transparency at X-junctions

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Abstract. Phenomenal transparency was studied in a stimulus geometry that differs markedly from the conventional Metelli configuration, namely four squares that abut at a common vertex. In case of subjective transparency one perceives either a bipartite square ground overlaid with a uniform transparent rectangle, or a uniform square background overlaid with a pair of mutually orthogonal, uniform, transparent rectangular regions. Thus, the generic interpretations are limited to "left", "right", "lower", or "upper" transparent rectangles or (when no subjective transparency occurs) "mosaic". All transparent cases are congruent, whereas the Metelli configuration allows distinct Gestalt interpretations. This avoids interactions between Gestalt factors and subjective transparency per se. Formal analysis reveals that—as in Metelli's case—a number of ambiguous cases (eg "left" or "lower") are to be expected. In the experiment we included these ambiguous cases as additional response categories. Observers who differ markedly on the Metelli configuration are virtually indistinguishable under the quad-square configuration. Moreover, observers reliably categorise the ambiguous instances as such; thus, "multiple transparency" has to be reckoned with as a bona fide percept.

1 Introduction

Phenomenal transparency (Metzger 1955; da Pos 1989) is the phenomenon in vision where a polygonal mosaic of uniform areas is perceived as consisting of fewer components than the individual tiles because some aggregates of tiles are perceived as *single*, *uniform* areas through which other features are seen. The part appears as a translucent overlay. This happens even if the individual tiles are *actual* parts, eg pieces of coloured paper. This is why the effect is so striking to naive observers when shown simple demos. The paradigmatic example is the configuration introduced by Metelli (1974a) (figure 1a) and widely studied as a paradigmatic case of transparency. If the luminance levels are assigned appropriately, observers are likely to experience a translucent centre square that overlays a bipartite, larger square background. If this is the perception, then the *four* polygons that tile the large square are perceived as two abutting rectangles overlaid by a single small square, thus a mere *three* polygons. The centre square appears to be of "one piece", cut from a single translucent sheet. If the luminances are assigned differently the pattern may appear as a "mosaic" of four abutting polygons. In the classical literature the former case is labeled "transparent", the latter "not transparent". Much of the literature is about the conditions that accompany the "experience of transparency" as opposed to the perception of a mosaic of opaque pieces (Masin 1997; Metelli 1974b; Singh and Anderson 2006).

The topic of phenomenal transparency has received much attention in recent literature. Most of these studies go far beyond the modest aims of this paper, though (Adelson and Anandan 1990; Albert 2008; Anderson 1997, 2003; Anderson et al 2008; Fleming and Anderson 2003; Masin 1997; Masin et al 2007; Singh 2004; Singh and Anderson 2002a, 2002b; Singh and Hoffman 1988). This paper focuses merely on the importance of Gestalt

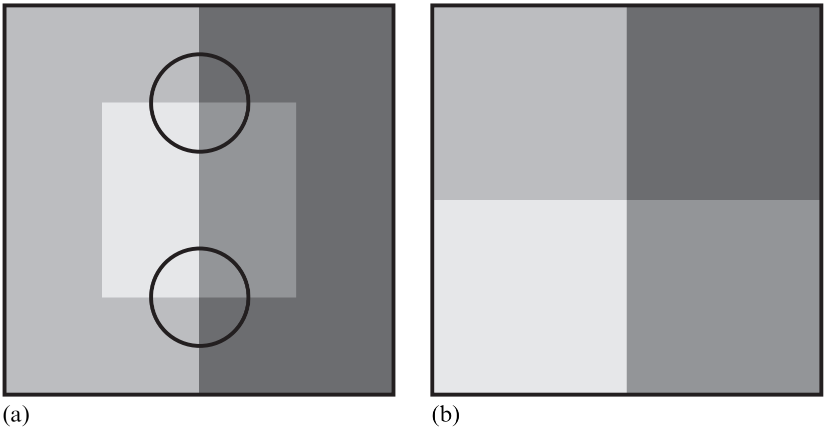


Figure 1. (a) The conventional Metelli configuration. The centre square can be seen as a translucent overlay over a bipartite larger square. The key information is provided by the two circled X-junctions. Notice that these are each other's mirror images; thus all the information is already present in one of them, the upper X-junction, say. Isolating this X-junction leads to configuration (b). This changes the response category "square" to "lower" and that of "hole" to "upper", whereas the categories "left" or "right" are retained. This is a great simplification, since "upper", "lower", "left", and "right" are equivalent up to isometries, whereas "square", "hole", and "left" or "right" are qualitatively distinct.

factors on the Metelli configuration. The work with the closest relation to ours is perhaps that by Kitaoka (2005). We address the topic via a specific reduction of the Metelli configuration, compared with results for the full Metelli task for the same group of observers.

In a previous publication (Koenderink et al 2008) we have shown that certain conditions formulated by Metelli indeed account for virtually all our data, but only if certain additional constraints are satisfied. The Metelli conditions (Metelli 1970) can be shown to be low-order approximations to the predictions of physics (Kubelka 1954; Kubelka and Munk 1934) of stacks of turbid sheets illuminated from the viewing direction. The formalism predicts the observations in detail when two additional conditions are met:

(i) the response categories should be sufficient. A mere "transparent" versus "not transparent" does not suffice;

(ii) the black and white levels should be visually "anchored".

Condition (i) is crucial and is closely tied to the geometry of the Metelli configuration. Investigation of the generic Gestalt interpretations reveals that there exist five response categories instead of the conventional two ["(transparent) square" and "mosaic"]. The additional categories are:

"hole"—the impression is of a large translucent sheet with a small square aperture that overlays a bipartite background and "left" or "right"—the impression of a vertically extended translucent rectangle that covers a background consisting of a large square with smaller centre square of different luminance, on either the left or the right.

The former ("hole") experiences have been reported by us in a previous communication (Koenderink et al 2008). The latter ("left" or "right") experiences have been reported earlier (Beck and Ivry 1988), though they don't figure prominently in the literature.

One author (Kitaoka 2005) introduced an even larger number of response categories, but (because several were not generic Gestalts) observers failed to use several of these. Kitaoka's "full layer transparency" is a complicated configuration that might be said to include the hole as a special case (though not explicitly mentioned by the author).

In our analysis (Koenderink et al 2008), this "full layer" response category is not generic because it can be shown to be equivalent to certain simpler configurations. Possibly Kitaoka's "case B" covers our "hole" response category; if this is so, this author's findings are similar to ours.

For each of the generic response categories one may work out the corresponding Metelli constraints. They are typically *distinct*, a fact that greatly affects both the experiments and their analysis.

The anchoring condition is also crucial. Formal analysis reveals that some configurations allow (at least) two possible interpretations and that scaling the white level may lead to a change of (predicted) category. Thus anchoring is necessary to define the stimulus unambiguously. This indicates that attempts to frame formal rules based upon ordinal relations of intensities (eg the well known work by Adelson and Anandan 1990) can only capture *part* of the constraints. It also indicates that it is highly desirable to provide anchoring in psychophysical experiments. Unfortunately, such anchoring is not implemented in the larger part of the literature.

As one reviewer of this paper observes, all this is common knowledge. Our failure to provide the relevant references is most likely due to a reluctance of previous authors to explicitly state the obvious. It is not a major aim of our study, though. In this paper we merely focus on the Gestalt factors as a crucial part of the Metelli task, and we have implemented explicit anchoring in the stimuli in order to ensure that the theoretical predictions apply to actual psychophysical data.

In previous experiments (Koenderink et al 2008) we found that observers use all five categories, not requiring additional ones, and that the formalism accounts for their responses in detail. Even *variations* in their responses can be understood through the vulnerability of the theoretical categorisations to perturbations of the luminance levels.

We encountered one striking exception, though. One of our observers (AD) essentially *never* used the "hole" category, putting "holes" (as predicted by the formalism) in the "mosaic" category. We speculated that this oddity (as compared to the other observers) might be due to Gestalt factors, rather than "phenomenal transparency" as such. Closer examination revealed a significant influence of Gestalt factors on the responses of the other observers too, though in no way as dramatic as the case of the "hole blind" observer. The fact that we accidentally discovered the "hole blindness" of observer AD was essentially a stroke of fortune. In the present paper we exploit this novel handle on the problem.

A check of this interpretation is possible only if one varies the stimulus configuration. The causally effective feature of the Metelli configuration with regard to phenomenal transparency is the X-junction (there are actually two of these, but they carry equivalent information; on the importance of X-junctions see Adelson and Anandan 1990). As suggested in figure 1b a mere tetrapartite area carries the same information without the Gestalt complications that accompany the Metelli configuration. In the configuration of figure 1b there are five Gestalt categories that may be denoted "left", "right", "lower", "upper", and "mosaic". The former four are congruent and thus effectively represent a single (qualitatively speaking) response category. There exist four ambiguous combinations such as "left-or-lower", and so forth. The nine response categories are shown in table 1.

In this paper we address two distinct issues. First, we run several generic and our "hole-blind" observer AD on the X-junction configuration in order to ascertain whether the exception is purely due to Gestalt factors or has to do with phenomenal transparency as such. Second, we investigate the ability of human observers to handle ambiguous cases (multiple response categories).

Table 1. The nine response categories.

mosaic	left	left or lower
	right	left or upper
	lower	right or lower
	upper	right or upper

2 Methods

Since the nine response categories (left, right, lower, upper, left-lower, left-upper, right-lower, right-upper, mosaic) do not occur with equal probabilities we used stratified sampling. We generated random luminance levels on $(0, 1)$ with the constraint that levels differed by at least 0.1 and retained equal numbers of the various categories. We used a set of 225 instances (25 per category) in total. Four observers ran a 9AFC task on this set, three times over, in randomised order. Use of the same set allows us to make various comparisons.

The stimulus configuration is shown in figures 2 and 3. Note the white and black squares that serve to anchor the white and black luminance levels *visually and at all times*. The observers indicate their choice by clicking on the appropriate button. Clicking a button immediately switches the display to the next stimulus. No feedback was provided at any time. Observers were left free to take all the time they needed, though in practice they used only roughly 6 s per setting. (As one reviewer of this paper justly remarked, in follow-up studies the issue of the temporal aspects may well turn out to be a rewarding object of attention.) The stimulus was presented on a linearised CRT monitor and viewed binocularly from 57 cm. Observers had normal or corrected-to-normal acuity. The basic square subtended 20 deg of visual angle.

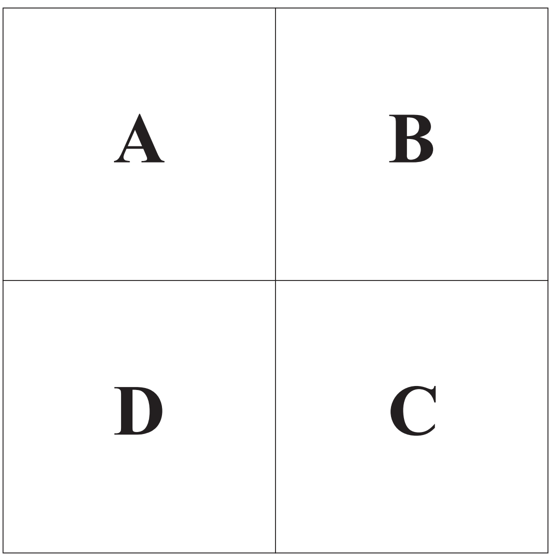


Figure 2. Definition of the areas in the stimulus configuration.

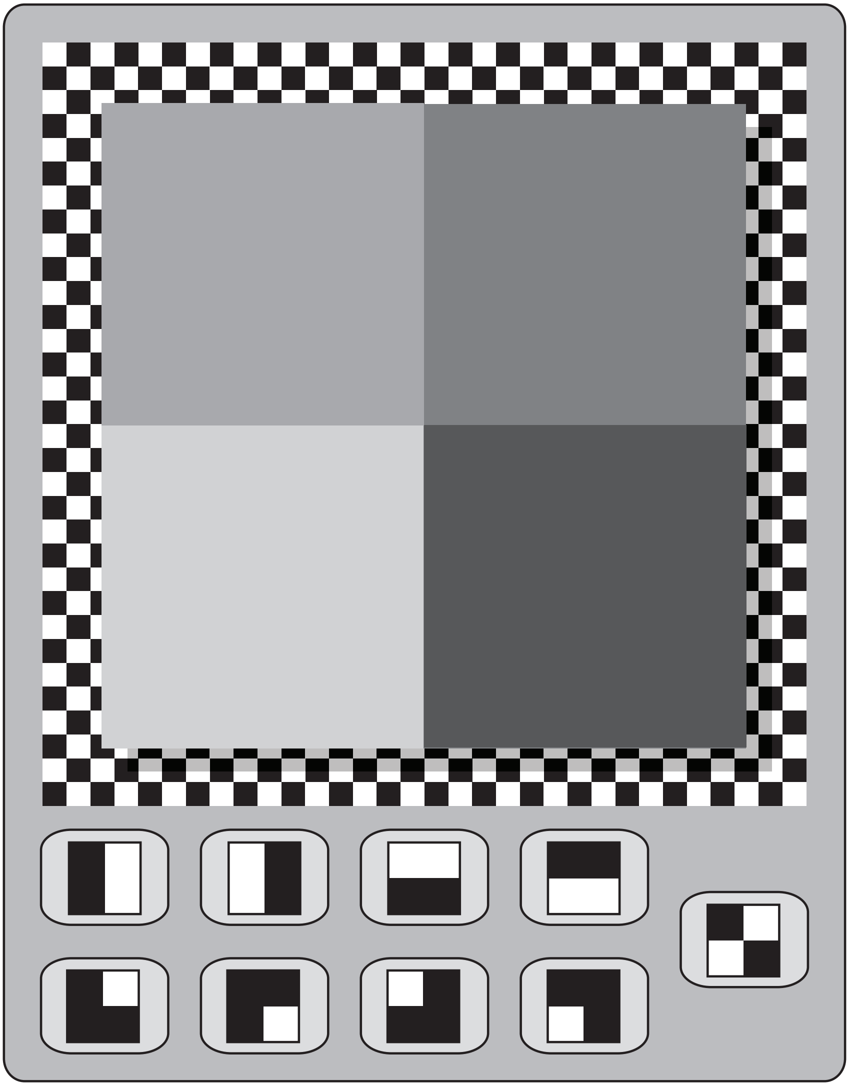


Figure 3. The stimulus configuration. Notice the black and white border that serves to anchor the black and white levels. The observer is required to select one of the response boxes: "left", "right", "lower", "upper", "mosaic", "left-or-lower", "left-or-upper", "right-or-lower", or "right-or-upper".

3 Results

Observers used all response categories, including those of the ambiguous cases. The appearance of the ambiguous stimuli varied. In some cases one sees initially a single transparent rectangle ("left", say) and thereafter discovers that it is also possible to see it as a different transparent rectangle ("lower", say). These interpretations invariably concerned mutually orthogonal rectangles, so that combinations like "left or right" or "lower or upper" never occurred. The two interpretations were seen successively, though the changeover never seemed to happen involuntarily as (for many observers) in the case of the familiar Necker cube. In other cases, one sees two orthogonal transparent rectangles overlapping each other and together overlapping a homogeneous square background. We have not pursued the precise stimulus conditions for the successive and simultaneous cases.

The analysis of the results is complicated owing to the presence of the ambiguous response categories. We define veridical responses in the sense of being predicted by the Metelli constraints. The various response types used in the analysis are explained in table 2.

Using this definition, all observers give a high percentage of veridical responses (response type 0); if we include partially correct responses (response type 4) in the count, the percentage correct is much higher still. Our analysis focuses upon the small fraction of instances that were categorised differently from the Metelli interpretation. These include "wrong assertions" (response type 1) such as "lower" instead of "mosaic", "misses" (response type 2) such as "mosaic" instead of "left", and "wrong categorisations" (response type 3) such as "lower" instead of "right".

Table 2. The response types (either fully or partially correct, wrong, etc) used in the analysis. A "correct response" implies the *exact* category; thus neither "left" nor "lower" counts as "correct" for "left or lower", only "left or lower" does. A "wrong assertion" means anything but "mosaic", if there is actually no transparency (thus "mosaic"). A "miss" means a "mosaic" response when there is actually transparency (anything but "mosaic"). A "wrong categorisation" means "left" instead of "upper", etc; thus "left" does not count as a "wrong categorisation" for "left or lower". A "partially correct categorisation" means "left" instead of "left or upper", for instance. Figure 7 lists all possibilities.

Category	Description
0	correct response
1	wrong assertion
2	miss
3	wrong categorisation
4	partially correct categorisation

A notion of the variability in observers' responses is obtained by simply counting the fraction of instances that were classified identically in the three sessions (remember that the same instances occur three times over). All observers gave three identical responses to more than half of the instances. The mean values are given in figure 4.

The answers may be broken down by response type. In figure 5a, the light-grey sector represents correct responses (response type 0), the medium-grey sector represents partially correct responses (response type 4), the dark-grey sector represents wrong assertions (response type 1), and the black sector represents errors (either wrong categorisations, response type 3 or misses, response type 2).

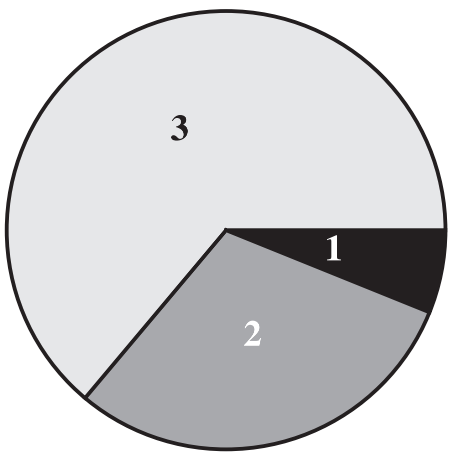
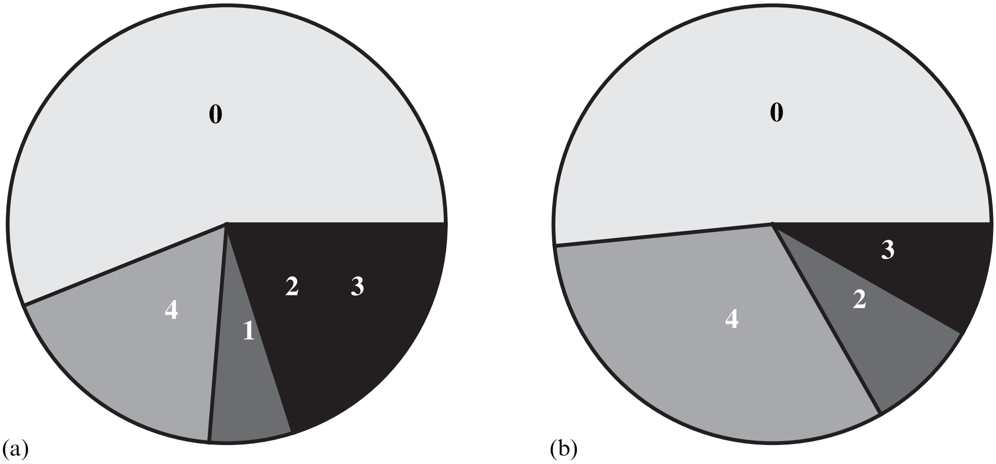


Figure 4. The number of repeated responses (out of 3 trials).



(a) (b)

Figure 5. (a) Responses broken down by type. The light-grey sector represents correct responses (response type 0), the medium-grey sector represents partially correct categorisations (response type 4), the dark-grey sector represents wrong assertions (response type 1), and the black sector represents misses and wrong categorisations (response types 2 and 3). (b) Ambiguous cases breakdown. The light-grey sector represents correct responses (response type 0), the medium-grey sector represents partially correct categorisations (response type 4), the dark-grey sector represents misses (response type 2), whereas the black sector represents wrong categorisations (response type 3).

The ambiguous cases (Metelli prediction of the mixed type) are special. In figure 5b the breakdown is limited to the ambiguous instances. The light-grey sector represents correct responses (response type 0), the medium-grey sector represents partly correct responses (response type 4), the dark-grey sector represents misses (response type 2), whereas the black sector represents category errors (response type 3). The amount of wrong categorisations (response type 3) is only 8%; thus observers are well able to handle the ambiguous stimuli.

The simplest analysis is based on the confusion matrix (figures 6 and 7, and table 3). Since the main diagonal is densely populated, it appears that responses are predominantly veridical, but the ambiguous cases are identified with less certainty than the non-ambiguous cases. Almost all confusions are due to either partially correct categorisations (response type 4) or misses (response type 2). Misses are very rare for the non-ambiguous instances, and somewhat more frequent for the ambiguous cases. Only very few wrong categorisations occur (response type 3), mainly for the ambiguous instances.

Since the categories are located in complicated regions of parameter space, a perturbation of the luminance levels will often affect the stimulus category. Some instances are located centrally in such a volume and will be relatively stable against such perturbations, whereas others will lie near a category boundary and be especially vulnerable. This "robustness" of the stimuli can be assessed by a simulation in which the luminance levels are randomly perturbed many times and the stimuli recategorised. Thus, one measures the probability of remaining in the original category, which is a useful measure of robustness. In figure 8 (lower graph) we plot the robustness against the rank order of robustness (the instances sorted by robustness). In the simulation, the levels were perturbed by Gaussian random noise with a standard deviation of 0.05. If we plot the empirically determined frequencies of correct responses for the same rank order, we obtain the upper graph. (Since four observers did three sessions each, there are only twelve discrete levels in this graph.) Notice that robustness implies frequent correct answers; the correlation between the graphs is 0.8. We conclude that the Metelli constraints also explain the larger part of the variation in the responses.

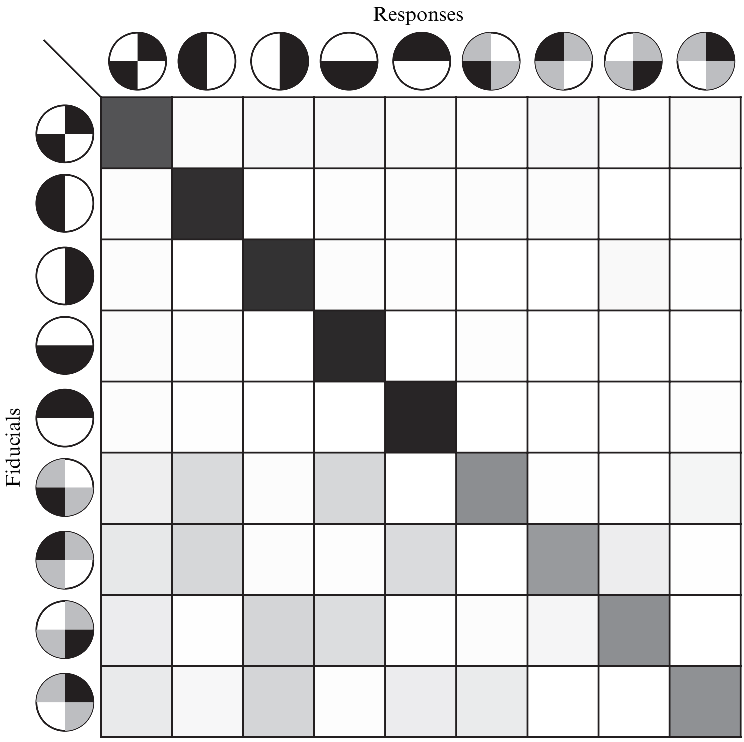


Figure 6. The average confusion matrix over all observers. Numerical values are specified in table 3.

	0	1	1	1	1	1	1	1	1
	2	0	3	3	3	4	4	3	3
	2	3	0	3	3	3	3	4	4
	2	3	3	0	3	4	3	4	3
	2	3	3	3	0	3	4	3	4
	2	4	3	4	3	0	3	3	3
	2	4	3	3	4	3	0	3	3
	2	3	4	4	3	3	3	0	3
	2	3	4	3	4	3	3	3	0

Figure 7. Meaning of the entries in the confusion matrix. Each row contains error categories for the given response; for instance, in the upper row one has the error categories for the "mosaic" response. It is a "correct response" (0) in the first column (stimulus actually a mosaic) and a "wrong assertion" (stimulus not a mosaic) for all other columns.

Table 3. Mean confusion matrix over all observers. The abbreviations used are: M, mosaic; L, left; R, right; l, lower; u, upper; Ll, left-lower; Lu, left-upper; Rl, right-lower; Ru, right-upper.

	M	L	R	l	u	Ll	Lu	Rl	Ru
M	82	2	3	4	2	1	3	1	2
L	1	95	0	1	1	1	1	0	0
R	1	0	94	1	1	0	0	2	0
l	2	1	0	97	0	1	0	0	0
u	1	0	0	0	98	0	0	0	1
Ll	7	16	1	18	0	54	0	0	4
Lu	10	18	1	1	16	0	47	7	0
Rl	8	0	18	15	0	1	4	53	0
Ru	9	3	19	1	8	9	0	0	52

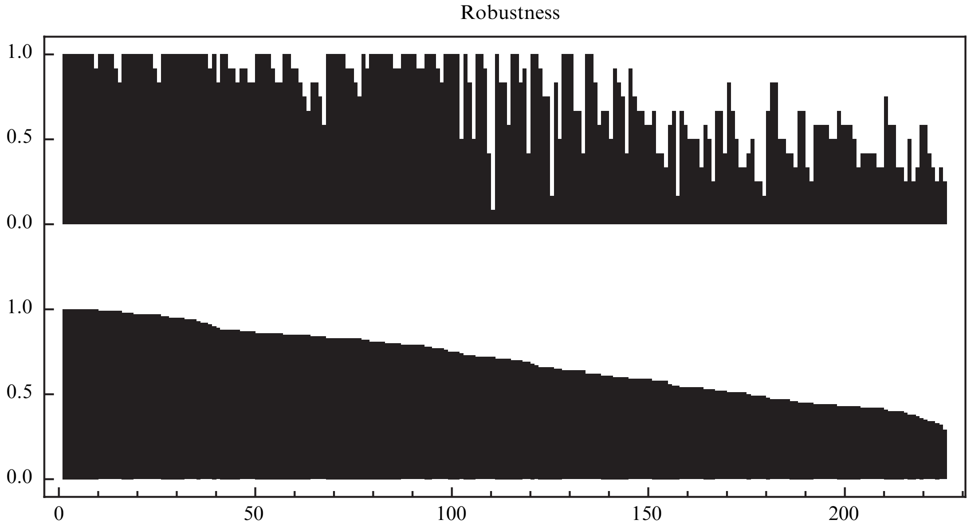


Figure 8. The robustness of the instances. The instances used in the experiment have been randomly perturbed many times and the probability of not changing category recorded. The instances have then been sorted by this probability. The rank order is on the abscissa, the probability on the ordinate of the lower graph. The upper graph shows the empirical frequencies of correct categorisations over all observers.

4 Discussion

With respect to our first objective our results are clear-cut. The anomalous observer AD who failed to use the "hole" category in the original Metelli task but put all such stimuli into the "mosaic" category does not differ in any respects from observers JK, MW, or SP in the present task (compare tables 3 and 4). Thus the anomalous behaviour of observer AD can be ascribed with certainty to the mixed nature of the Metelli task which does not solely address the issue of phenomenal transparency (as it is conventionally made out to be) but has a nontrivial and sometimes determining interaction with Gestalt factors, especially figure–ground segregation involving apertures rather than isolated objects.

With respect to the second objective the result is also pretty clear, though there are still some unresolved issues left. First of all, it is evident that human observers are aware of the ambiguous nature of certain instances and in many (better than 50%) of the cases classify them correctly (response type 0). In most (~70%) of the remaining instances observers go for at least one of the two valid interpretations (response type 4).

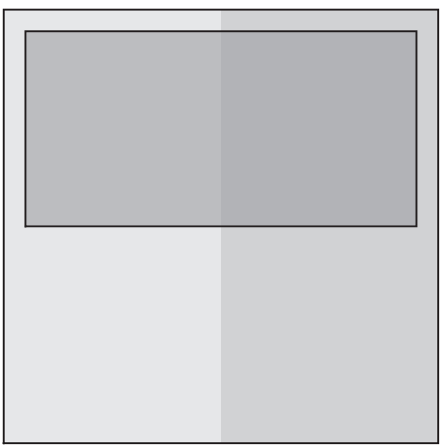
Of the remaining cases about half is missed (put in the default "mosaic" category, response type 2), the rest incorrectly classified (response type 3).

Phenomenologically there appear to be two qualitatively different percepts, though. For some instances one sees two different interpretations *successively*, the switchover being almost entirely through a voluntary shift of attention. For other instances one is aware of two *simultaneously* present, partially overlapping transparent rectangles, overlaid over a uniform (instead of bipartite) background. Since these impressions were unexpected, they were not distinguished through different response categories in our experiment.

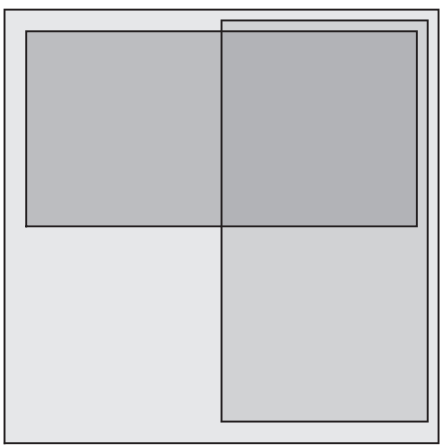
Table 4. The confusion matrix for observer AD. The abbreviations used are the same as in table 3. The confusion matrix for observer AD is not significantly different from the mean confusion matrix. A principal-component analysis reveals that the mean accounts for 96% of the variance, and that the projections on the mean are in the ratios AD : JK : MW : SP = 1.0075 : 0.9345 : 1.0567 : 1.0012. Thus, observer AD is in no way special.

	M	L	R	l	u	Ll	Lu	Rl	Ru
M	91	1	0	3	0	0	3	1	1
L	0	89	0	3	1	4	3	0	0
R	0	0	84	5	1	0	0	8	1
l	4	1	0	93	0	1	0	0	0
u	1	1	0	0	96	0	0	0	1
Ll	7	5	0	21	0	64	1	0	1
Lu	16	9	1	0	23	0	48	3	0
Rl	7	0	1	13	0	4	1	73	0
Ru	5	1	12	0	15	0	0	0	67

The simultaneous interpretation evidently describes a physical configuration that is not captured by the Metelli constraint used by us to define the notion of veridicality. In this case one has four regions (figure 9); one is due to the background alone, two are due to the two (different) rectangular overlays, and one is due to a sandwich of *three* layers, two of them transparent. In principle there exist two distinct interpretations of this type because either of the two transparent rectangles might lie in front.



(a)



(b)

Figure 9. (a) An interpretation with a bipartite, opaque background (large square) covered with a single, horizontal, rectangular transparent sheet. (b) An interpretation with three partially overlapping sheets, two of them translucent (the horizontal and vertical rectangles), the background (large square) opaque. Notice the ambiguity of the ordering of the transparent sheets.

Thus, it looks as if the ambiguous cases should really be analysed via three distinct constraints: (for the successive case) the standard Metelli constraint used in our experiment, the other two for the two interpretations discussed in this paragraph. A closer examination reveals that the formal constraints are identical, it is only the physical interpretation that is different. That human observers apparently distinguish such cases has therefore nothing to do with the Metelli constraints. We cannot be certain whether observers make these distinctions in a systematic way or merely randomly; this calls for further experiments.

Notice that the new interpretations may also occur in the case of the conventional Metelli configuration. The literature indeed provides several hints (Anderson 1997; Kitaoka 2005; Metelli et al 1981, 1985). One would then see a large uniform background, covered on one side by a vertical rectangular transparent sheet, the combination of these two (partially) covered with a small, transparent centre square. This seems different from Beck et al's (1984) "partial transparency" and might be called "multiple transparency". This case is just as generic as the partial transparency interpretation and there appears to be no reason why Gestalt factors would prevent its occurrence. It is formally not distinct from the ambiguous case "transparent centre square-left (or right) rectangle", though. Of course, this means that the perception of "transparency" would not simply be a consequence of certain photometric relations, but is dependent on a variety of different cues, a concept current in the modern literature (Anderson 2003; Fleming and Anderson 2003; Kitaoka 2005; Singh and Anderson 2002a, 2002b; Singh and Hoffman 1988). Anything that might play a role in perceptual segmentation probably enters in cases of "transparency".

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References

Adelson E H, Anandan P, 1990 "Ordinal characteristics of transparency", paper presented at the AAAI-90 Workshop on Qualitative Vision, 29 July 1990, Boston, MA

Albert M K, 2008 "The role of contrast in the perception of achromatic transparency: Comment on Singh and Anderson (2002) and Anderson (2003)" *Psychological Review* **115** 1127 – 1143

Anderson B L, 1997 "A theory of illusory lightness and transparency in monocular and binocular images: the role of contour junctions" *Perception* **26** 419 – 453

Anderson B L, 2003 "The role of occlusion in the perception of depth, lightness and opacity" *Psychological Review* **110** 762 – 784

Anderson B L, Singh M, O'Var J, 2008 "Natural decompositions of perceived transparency: Reply to Albert (2008)" *Psychological Review* **115** 1144 – 1153

Beck J, Ivry R, 1988 "On the role of figural organization in perceptual transparency" *Perception & Psychophysics* **44** 585 – 594

Beck J, Prazdny I, Ivry R, 1984 "The perception of transparency with achromatic colors" *Perception & Psychophysics* **35** 407 – 422

Fleming R W, Anderson B L, 2003 "The perceptual organization of depth", in *The Visual Neurosciences* Eds L Chalupa, J S Werner (Cambridge, MA: MIT Press) pp 1284 – 1299

Kitaoka A, 2005 "A new explanation of perceptual transparency connecting the X-junction contrast-polarity model with the luminance-based arithmetic model" *Japanese Psychological Research* **47** 175 – 187

Koenderink J J, Doorn A J van, Pont S C, Richards W A, 2008 "Gestalt and phenomenal transparency" *Journal of the Optical Society of America A* **25** 190 – 202

Kubelka P, 1954 "New contributions to the optics of intensely light-scattering materials, Part II. Non-homogeneous layers" *Journal of the Optical Society of America* **44** 330 – 334

Kubelka P, Munk F, 1934 "Ein Beitrag zur Optik des Farbanstriche" *Zeitschrift für Technische Physik* **12** 593 – 601

Masin S, 1997 "The luminance conditions of transparency" *Perception* **26** 39 – 50

Masin S C, Tommasi M, Pos O da, 2007 "Test of the Singh – Anderson model of transparency" *Perceptual and Motor Skills* **104** 1367 – 1374

Metelli F, 1970 "An algebraic development of the theory of transparency" *Ergonomics* **13** 59 – 66

Metelli F, 1974a "The perception of transparency" *Scientific American* **230**(4) 91–98
Metelli F, 1974b "Achromatic color conditions in the perception of transparency", in *Perceptions: Essays in Honor of J. J. Gibson* Eds R B MacLeod, H L Pick (Ithaca, NY: Cornell University Press) pp 96–116
Metelli F, Masin S C, Manganelli M, 1981 "Partial transparency" *Atti dell'Accademie Patavina di Scienze, Lettere ed Arti* **92** 115–169
Metelli F, Pos O da, Cavedon A, 1985 "Balanced and unbalanced, complete and partial transparency" *Perception & Psychophysics* **38** 354–366
Metzger W, 1955 "Über Durchsichtigkeits-Erscheinungen" *Rivista di Psicologia, Fascicolo Giubilare* **49** 187–189
Pos O da, 1989 *Trasparenze* (Padua: Icone)
Singh M, 2004 "Lightness constancy through transparency: internal consistency in layered surface representations" *Vision Research* **44** 1827–1842
Singh M, Anderson B L, 2002a "Toward a perceptual theory of transparency" *Psychological Review* **109** 492–519
Singh M, Anderson B L, 2002b "Perceptual assignment of opacity to translucent surfaces: The role of image blur" *Perception* **31** 531–552
Singh M, Anderson B L, 2006 "Photometric determinants of perceived transparency" *Vision Research* **46** 879–894
Singh M, Hoffman D D, 1988 "Part boundaries alter the perception of transparency" *Psychological Science* **9** 370–378

Appendix: The basic Metelli constraint

Consider the stimulus configuration of figure 2. Let the luminance levels of the areas A, B, ... be denoted as a, b, ... Then the condition for a transparent rectangle on the right side (area $\mathrm{B} \cup \mathrm{C}$) is

$$((b > c) \wedge (a > d) \wedge (((b < a) \wedge (bd < ac)) \vee ((a + c + bd > b + ac + d) \wedge (b \geqslant a)))) \vee$$
$$((b < c) \wedge (a < d) \wedge (((bd > ac) \wedge (b < a)) \vee ((b \geqslant a) \wedge (a + c + bd < b + ac + d)))) \,.$$

The constraints for transparent rectangles on the left (that is the area $\mathrm{A} \cup \mathrm{D}$), the lower part (the area $\mathrm{C} \cup \mathrm{D}$), or the upper part (the area $\mathrm{A} \cup \mathrm{B}$) can be obtained by appropriate permutations of the levels $\{a, b, c, d\}$. The constraints overlap, leaving room for the ambiguous combinations left-upper, left-lower, right-upper, and right-lower. When none of the constraints is satisfied, the default category "mosaic" applies.

Straightforward integration yields the probabilities of the various categories when the levels are randomly drawn from a uniform distribution on [0, 1]. The non-ambiguous categories left, right, lower, or upper occur with probability 1/12, the ambiguous categories with probability 1/24, and the default mosaic case with probability 1/2.

The constraints are invariant against inversion of the levels, that is to say, against the substitution of $a \rightarrow (1 - a)$, and so forth. They are not invariant against scaling, that is to say, the substitution $a \rightarrow \lambda\, a$, and so forth. Thus the white level anchoring is a necessary requirement.

The mere order of levels is sufficient to settle the unambiguous left, right, lower, and upper cases (two possible orders for each), and some of the mosaic cases (either possible orders), but there are eight orders that allow triples of interpretations such as (left-upper, mosaic, right-lower). This is an important observation because it has been suggested (Adelson and Anandan 1990) that human observers might use the order of luminance levels as the effective cue. Perhaps unfortunately, our results contradict this attractive hypothesis.

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